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How to find quickly good solutions and tight bounds on their quality for hard combinatorial optimization and integer programming problems. One of the basic ideas is to do repeated local search by solving small integer programs defined by fixing many of the variables. We demonstrate the effectiveness of such an approach by developing and testing such an algorithm for large-scale fixed-charge network flow (FCNF) problems.

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COMBINING EXACT AND HEURISTIC APPROACHES FOR DISCRETE OPTIMIZATION

AFOSR Contract FA9550-09-1-0061

Final Report

February 15, 2009 – November 30, 2011

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A primary focus of this project is on how to find quickly good solutions and tight bounds on their quality for hard combinatorial optimization and integer programming problems. One of the basic ideas is to do repeated local search by solving small integer programs defined by fixing many of the variables. We demonstrate the effectiveness of such an approach by developing and testing such an algorithm for large-scale fixed-charge network flow (FCNF) problems [1]. The solution approach combines mathematical programming algorithms with heuristic search techniques. To obtain high-quality solutions, it relies on local search with carefully chosen neighborhoods derived from the arc-based formulation of FCNF. To obtain lower bounds, the linear programming relaxation of the path-based formulation of FCNF is used and strengthened with cuts discovered during the neighborhood search. The neighborhood selection process incorporates randomization to diversify the search and learning to intensify the search. The resulting solution approach is very effective. For instances with 500 nodes, with 2,000, 2,500, and 3,000 arcs, and with 50, 100, 150, and 200 commodities, we compared the quality of the solution produced by our approach with the best solution found by CPLEX after 15 minutes of computation and after 12 hours of computation. On average, the solution we found in less than 15 minutes is 35% better than CPLEX’s best solution after 15 minutes and 20% better than CPLEX’s best solution after 12 hours. Furthermore, we find a better solution than CPLEX’s best solution after 15 minutes within 1 minute, and CPLEX’s best solution after 12 hours within 3 minutes. We also compared the quality of the solutions produced by our approach with the quality of the solutions produced by a recent implementation of a tabu search algorithm. For nearly all instances in their test set, our solution is better and is found much faster.

A major accomplishment in the final year of this project has been the substantial generalization of our previous work in [1 the solution of small integer programs in a heuristic search scheme. The new approach [2] retains the strengths of integer programming based heuristic search methods, i.e., problem structure can be exploited to produce high-quality solutions quickly, but remedies their main weaknesses, i.e., having to design problem specific neighborhoods and being unable to provide performance guarantees. Moreover, while the method benefits significantly from the use of structure, it does not formally require structure. The approach uses an extended formulation of the problem whose solution automatically and dynamically yields a restricted

integer program to be solved next. The extended formulation is solved with a branch-and-price algorithm, which, when run to completion, produces a provable optimal solution. However, it gives dual bounds throughout the execution, so even if terminated prematurely, it provides a performance guarantee. We apply the approach to the multi-commodity fixed-charge network flow problem (MCFCNF). We demonstrate the effectiveness of the approach by comparing its performance on instances of the MCFCNF against both an exact solver, CPLEX, and an integer programming based heuristic search method. The primal solution found by the proposed approach in 30 minutes is often optimal, better than the one produced by the integer programming based heuristic search method in the same time frame, and better than the one CPLEX produced in 6 hours.

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